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# Diffusion Enhances Spontaneous Electroweak Baryogenesis

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We include the effects of diffusion in the electroweak spontaneous baryogenesis scenario and show that it can greatly enhance the resultant baryon density, by as much as a factor of  $1/\alpha_w^4 \sim 10^6$  over previous estimates. Furthermore, the baryon density produced is rather insensitive to parameters characterizing the first order weak phase transition, such as the width and propagation velocity of the phase boundary.

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## 1. Introduction

The scenario of electroweak baryogenesis (EWB) [1] owes its appeal to the fact that—unlike most alternatives—it involves particles and interactions known to exist, and may be experimentally testable. (For a relatively recent review, see [2].) Baryogenesis at the weak scale is made possible by the fact that in the symmetric phase at temperature  $T$ , baryon violation is expected to occur at a rate  $\Gamma \sim \alpha_w^4 T$ , where  $\alpha_w$  is the weak fine structure constant [3]. Such theories require a first order weak phase transition in order to provide the departure from thermal equilibrium necessary for baryogenesis [4]. During the phase transition, bubbles of broken  $SU_L(2) \times U_Y(1)$  phase nucleate and rapidly expand, and the source of nonequilibrium physics necessary for baryogenesis is the interaction between particles in the plasma and the expanding bubble wall that separates the phases. The greatest challenge for an EWB mechanism is reconciling the fact that nonequilibrium  $CP$ -violating physics is occurring within the bubble wall itself, while baryon violation is only rapid outside the bubble in the symmetric phase (and perhaps in the leading edge of the wall). This difficulty is easily surmounted by the charge transport mechanism [5-8] in the limit when the bubble wall is narrow compared to particle mean free paths. With this mechanism  $CP$  violation in the phase boundary leads to the creation of net global charges among the particles reflecting off the domain wall; the charge is then transported into the region in front of the advancing domain wall. The global charge distributions in front of the wall contain a nonzero density of  $SU_L(2)$  doublet fermions, and therefore bias anomalous baryon number fluctuations toward production of baryon number <sup>1</sup>.

In the opposite limit, when the width of the bubble wall is large compared to the mean free path of fermions in the plasma, the fermions see an adiabatically changing background Higgs field and are not reflected. Thus the charge transport regime does not apply, and alternative analyses have been proposed to explain how EWB would proceed within the wall itself. These analyses are based on the idea of “spontaneous baryogenesis”, where a time varying background  $CP$ -odd field arising from spontaneous symmetry breakdown provides the necessary  $CPT$  violation to bias  $B$  violating interactions toward the creation

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<sup>1</sup> An alternative mechanism is proposed in [9], in which nonzero Higgs winding number is produced at the bubble wall, subsequently decaying in its wake, producing baryon number.

of net baryon number [10]. For EWB it is a nonzero  $CP$  violating phase in a Higgs field that plays the role of the background field and leads to a local excess of  $SU_L(2)$  doublet anti-fermions, causing anomalous electroweak processes to be biased in favor of producing baryon number within the bubble wall [11-13]. The problem with this scenario is that as the Higgs field turns on, baryon violating interactions turn off exponentially fast, the rate being proportional to  $\exp(-2M_W/T)$ . Taking the value of the Higgs vev where the baryon violation “cuts off” to be  $\phi_{co}$ , it has been argued that one can take  $\phi_{co}$  to be as large as  $\phi_{co} \simeq 14\alpha_w T/g$ , where  $g$  is the weak coupling constant [14], in which case the resultant baryon asymmetry is proportional to a factor of  $\alpha_w^4 \simeq 10^{-6}$  intrinsic to anomalous baryon violation. However, Dine and Thomas have argued [15] that in fact  $\phi_{co} \simeq \alpha_w T/g$ , and that as a consequence the resultant baryon asymmetry is suppressed by an additional factor of  $\alpha_w^4$  in the two-Higgs model, or  $\alpha_w^2$  in the minimal supersymmetric model [13]—suppression factors too small to account for the observed asymmetry  $n_B/s \simeq 10^{-10}$  when one includes small  $CP$  violating angles. The correct value for  $\phi_{co}$  is currently not known, and so the Dine–Thomas objection throws in doubt the scenario of EWB in the spontaneous baryogenesis (thick wall) regime. This is disturbing, since models of the electroweak sector that do not involve  $SU_L(2)$  singlet fields generically produce wide domain walls during a first order phase transition, and therefore rely on the spontaneous baryogenesis mechanism [16,14].

In this paper we consider the role of particle diffusion for EWB in the spontaneous baryogenesis regime. The potential importance of diffusion was first pointed out by Joyce, Prokopec and Turok [17], who concluded that diffusion was inimical to spontaneous baryogenesis. In the present work we arrive at a different conclusion—that diffusion moves the asymmetry in  $SU_L(2)$  doublet fermions from within the bubble wall into the symmetric phase in front, where baryon violation is not exponentially suppressed. As a result the Dine–Thomas suppression factors are evaded, and the spontaneous baryogenesis scenario with diffusion ends up looking remarkably like the charge transport scenario [5,6].

We show that not only can spontaneous electroweak baryogenesis account for the observed baryon asymmetry of the universe, but that the asymmetry produced is not very sensitive to the value one takes for  $\phi_{co}$ , or to features of the weak phase transition, such

as the width of the bubble wall (assuming it to be sufficiently wide), or its velocity.

By analyzing the dynamical equations associated with spontaneous baryogenesis we are also able to treat a second important source of suppression recently discussed by Giudice and Shaposhnikov [18]. They pointed out that when all of the quarks are treated as massless, QCD sphaleron processes equilibrate to zero the same charge densities that serve to drive electroweak baryogenesis. We find evidence that QCD and electroweak sphalerons compete, but that even in the massless quark approximation a nonzero baryon density results, which is surprisingly insensitive to the overall rate of sphaleron processes, being proportional to the ratio of electroweak to strong sphaleron rates. If this effect persists in a more sophisticated treatment including finite quark mass effects then one of the greatest uncertainties about electroweak baryogenesis — the overall rate of sphaleron processes — is eliminated.

## 2. Diffusion equations in the two Higgs doublet model

To be specific, we will focus on the two Higgs doublet model with non-Kobayashi-Maskawa  $CP$  violation as a model for electroweak baryogenesis [9], for which the spontaneous baryogenesis mechanism is described in [12]. During a first order phase transition bubbles of the broken phase will nucleate and expand. As a top quark in the plasma traverses the bubble wall, it interacts with the background Higgs field through its Yukawa coupling  $\lambda_t \approx 1$ . In the adiabatic limit of a slowly moving or broad domain wall, the top quark has a spacetime dependent mass term of the form  $m_t = \lambda_t H(\mathbf{r}, t) e^{-i\theta(\mathbf{r}, t)}$ , where the spacetime dependent phase  $\theta$  arises from  $CP$  violation in the Higgs potential. The effects of  $\theta$  are most easily analyzed by performing a hypercharge rotation of all the fields in the theory, removing the  $\theta$  dependence from the top quark mass term, at the expense of inducing an interaction of the form

$$-2\partial_\mu \theta(\mathbf{r}, t) J_Y^\mu(\mathbf{r}, t) . \quad (2.1)$$

With nonzero  $\mu \equiv -2\dot{\theta}$ , we see that this interaction resembles a thermodynamic charge constraint, with  $\mu$  being the chemical potential. Since hypercharge is not conserved inside the domain wall, we refer to the dynamically generated  $\mu$  as a charge potential, to

distinguish it from an imposed constraint on a conserved charge. Exploiting the resemblance, we know that the free energy is minimized by having nonzero particle densities  $n_i = k_i y_i \mu T^2 / 6$ , where  $y_i$  is the hypercharge, and  $k_i$  a statistical factor that differs for fermions and bosons. In the limit that all fermions are massless except the top quark, the interaction (2.1) leads to production of  $q \equiv (t_L, b_L)$ ,  $t_R$  and  $H$  particles; if  $\mu$  is negative, then the excess of  $q$  doublets (with  $Y = +1/6$ ) is also negative, and anomalous electroweak processes are biased to produce baryon number <sup>2</sup>.

To see whether diffusion can be a significant effect, it is useful to consider the dimensionless quantity

$$\epsilon_D \equiv D / L_w v_w ,$$

where  $D$  is the diffusion constant for quarks,  $L_w$  is the width of the domain wall, and  $v_w$  is the wall velocity. None of these quantities is known accurately, with estimates for  $v_w$  ranging from  $0.1c$  to  $c/\sqrt{3} \simeq 0.6c$ ;  $L_w$  ranging from  $10/T$  to  $40/T$ , and  $D$  (classically equal to  $\ell c/3$ , where  $\ell$  is the mean free path) ranging from  $1/T$  to  $5/T$ . Evidently  $\epsilon_D$  can be  $\mathcal{O}(1)$ , and hence one may expect significant (*i.e.*  $\mathcal{O}(1)$ ) redistributions of particle densities due to diffusion. As we will show, this eliminates the sensitive dependence on the value of  $\phi_{co}$  discussed by Dine and Thomas [15].

In order to understand the detailed effect of the interaction (2.1), we first derive the diffusion equation relevant for a single particle species with local number density  $j^0 \equiv n$  and number current  $\vec{J}$  interacting with a slowly varying background field  $A_0$  and  $\vec{A}$  through the interaction

$$\mathcal{L}_{int} = A_\mu j^\mu . \tag{2.2}$$

We assume that  $j^\mu$  is not exactly conserved, since  $j^\mu$  is to eventually play the role of the hypercharge current, which is violated in the broken phase. Following the spirit of the usual derivation of Fick's law and the diffusion equation, we perform a simultaneous expansion to first order in the deviations from equilibrium ( $n - n_0$ ), in derivatives of ( $n - n_0$ ) and of the background field  $A$ , and in the particle number violating interactions. Keeping only

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<sup>2</sup> Hypercharge screening proves not to affect the result [19,7]; we will return to this point later.

leading terms, we find

$$\begin{aligned}\dot{n} &= -\vec{\nabla} \cdot \vec{J}[A, n] - \frac{\gamma}{T} \frac{\partial F}{\partial Q} \\ &= -\vec{\nabla} \cdot \vec{J}[A, n] - \Gamma \frac{(n - n_0[A])}{k} ,\end{aligned}\tag{2.3}$$

where  $F$  is the free energy,  $\gamma$  is the violation rate per unit volume of the charge  $Q = \int d^3x n$ , and we have defined the rate  $\Gamma$  as

$$\Gamma \equiv \frac{6\gamma}{T^3} .\tag{2.4}$$

The parameter  $k$  appearing in (2.3) is a statistical factor,

$$k = (\text{number of spin degrees of freedom}) \times \begin{cases} 2 & \text{bosons} \\ 1 & \text{fermions} \end{cases}\tag{2.5}$$

up to small corrections due to thermal particle masses. It remains to find the constitutive relations for  $\vec{J}$  and  $n_0$ . Using rotational covariance we find

$$\begin{aligned}\vec{J} &= -D\vec{\nabla}n + a_1\vec{A} + a_2\partial_t\vec{A} + a_3\vec{\nabla}A_0 + \dots \\ n_0 &= b_1A_0 + \dots ,\end{aligned}\tag{2.6}$$

where the ellipses refer to higher powers of derivatives acting on  $n$  and  $A$ , as well as terms involving the particle number violating interactions. The coefficients  $a$  and  $b$  are assumed to roughly equal the appropriate powers of the mean free path  $\ell$  dictated by dimensional analysis, so that the derivative expansion is justified for hydrodynamic modes of wavelength much longer than  $\ell$ . (The size of the modes we wish to consider are set by the bubble wall width  $L_w$  assumed to satisfy  $L_w \gg \ell$ , which is the spontaneous baryogenesis regime.)

If the current is conserved and  $A_\mu$  is a total divergence, then the interaction (2.2) has no physical effect. From this we deduce that

$$a_1 = 0 , \quad a_3 = -a_2 .$$

It does not follow that  $b_1 = 0$ , since this coefficient only appears in eq. (2.3) proportional to the current nonconserving effect  $\Gamma$ . We can fix  $b_1$ , however, by considering the simple case  $A_0 = \mu$  and  $\vec{A} = 0$ . The minimum of the free energy is then at  $n = k\mu T^2/6$ , and so that must be the value  $n_0$  in eq. (2.3). It follows that the diffusion equation relevant for the interaction (2.2) is

$$\dot{n} = D\nabla^2(n - \phi) - \Gamma \frac{(n - n_0)}{k} ,$$

where

$$D\nabla^2\phi \equiv a_2\vec{\nabla} \cdot (\partial_t\vec{A} - \vec{\nabla}A_0) , \quad n_0 = kA_0T^2/6 .$$

This result can be readily taken over to the interaction of interest in eq. (2.1), and generalized to many particle species. Since  $A_\mu = -2\partial_\mu\theta$  it follows that  $\phi$  in the above equation vanishes, and that

$$\dot{n}_i = D_{ij}\nabla^2 n_j - \Gamma_{ij} \left( \frac{n_j + k_j y_j \dot{\theta} T^2/3}{k_j} \right) , \quad (2.7)$$

where  $n_i$  is the density of particle species  $i$ ,  $y_i$  is its hypercharge, and  $k_j$  its statistical factor. The diffusion matrix  $D_{ij}$  has diagonal elements whose magnitudes are given by the particle mean free paths, while the off-diagonal elements are smaller by at least one power of  $\alpha$ , the fine structure constant of the transition interaction. We will work to leading order in perturbation theory, only keeping the diagonal entries:

$$D_{ij} = D_i\delta_{ij} + \mathcal{O}(\alpha) .$$

As for the  $\Gamma_{ij}$ —hypercharge is only broken by the vev of the Higgs field, and so in the limit that we ignore weak mixing angles, all Yukawa couplings except for the top quark's, and anomalous strong interactions, the only species of particles whose densities are affected by a nonzero  $\dot{\theta}$  are the left handed third family doublet denoted by  $q \equiv (t_L + b_L)$ , the right handed top quark  $t \equiv t_R$ , and the Higgs particles  $h \equiv (h_1^- + h_1^0 + h_2^- + h_2^0)$ . The individual particle numbers of these species can change through the top quark Yukawa interaction, the top quark mass, the Higgs self interactions, and anomalous weak interactions, at the rates  $\Gamma_y$ ,  $\Gamma_m$ ,  $\Gamma_h$  and  $\Gamma_{ws}$  respectively.

Including strong sphalerons (with a rate  $\Gamma_{ss}$ ) would allow the generation of right handed bottom quarks,  $b \equiv b_R$ , as well as first and second family quarks. However since strong sphalerons are the only processes which generate significant numbers of first and second family quarks, and all quarks have approximately the same diffusion constant, these densities are constrained algebraically in terms of  $b$  to satisfy

$$q_{1L} = q_{2L} = -2u_R = -2d_R = -2s_R = -2c_R = -2b_R \equiv -2b . \quad (2.8)$$

To simplify the equations we may use the fact that the only charge potential generated is for hypercharge, and thus each  $n_0$  is proportional to  $k$  times the hypercharge for the species—this allows us to eliminate all  $n_0$  in favor of  $h_0$ . Using the relations

$$3q_0/k_q + l_0/k_l \propto (3y_q - y_l) = (1/6)3 - 1/2 = 0$$

$$(q_0/k_q - h_0/k_h - t_0/k_t) \propto (y_q - y_h - y_t) = (1/6) - (-1/2) - (2/3) = 0$$

we see that the source terms proportional to  $\dot{\theta}$  drop out of  $\Gamma_y$ ,  $\Gamma_{ws}$  interactions and are only present proportional to the hypercharge violating  $\Gamma_m$  and  $\Gamma_h$ , consistent with the arguments of Dine and Thomas [15].

Eq. (2.7) may now be written as

$$\begin{aligned} \dot{q} &= D_q \nabla^2 q - \Gamma_y [q/k_q - h/k_h - t/k_t] - \Gamma_m [q/k_q - t/k_t - h_0/k_h] \\ &\quad - 6\Gamma_{ws} [3(q - 4b)/k_q + l/k_l] - 6\Gamma_{ss} [2q/k_q - t/k_t - 9b/k_b] \\ \dot{t} &= D_t \nabla^2 t - \Gamma_y [-q/k_q + h/k_h + t/k_t] \\ &\quad - \Gamma_m [-q/k_q + t/k_t + h_0/k_h] + 3\Gamma_{ss} [2q/k_q - t/k_t - 9b/k_b] \\ \dot{h} &= D_h \nabla^2 h - \Gamma_y [-q/k_q + t/k_t + h/k_h] - \Gamma_h (h - h_0)/k_h \\ \dot{b} &= D_b \nabla^2 b + 3\Gamma_{ss} [2q/k_q - t/k_t - 9b/k_b] \end{aligned} \tag{2.9}$$

where the  $k$  factors are given by (2.5)

$$k_q = 6, \quad k_t = 3, \quad k_h = 8, \quad k_b = 3. \tag{2.10}$$

Several simplifications of equations (2.9) can be made. First we ignore the curvature of the bubble wall, and so  $\Gamma_m$ ,  $\Gamma_h$ , and  $\Gamma_{ws}$  are only functions of  $z \equiv |\vec{r} - \vec{v}_w t|$ , where  $\vec{v}_w$  is the bubble wall velocity. Explicitly, we assume that  $\Gamma_m$ , and  $\Gamma_h$  vary as the square of the Higgs vev, while  $\Gamma_{ws}$  is approximated by a step function:  $\Gamma_{ws}(z) \simeq \theta(z - z_{co})\Gamma_{ws}$ . The parameters  $D_t$ ,  $D_h$ ,  $\Gamma_y$  and  $\Gamma_{ss}$  are taken to be spacetime constants. We will assume that the density perturbations of interest are functions of  $z$  and  $t$  only. We will also take  $D_q \simeq D_t \simeq D_b$ , which is correct up to order  $\alpha_w/\alpha_s$ . Then, ignoring  $\mathcal{O}(\Gamma_{ws}^2)$  effects on the final baryon density, we may set  $b = -(q + t)$  and eliminate the  $b$  equation; furthermore,



since no leptons are produced in the limit that we ignore lepton Yukawa couplings we may set  $l = 0$ .

With these assumptions we arrive at the diffusion equations for  $q(z, t)$ ,  $t(z, t)$ , and  $h(z, t)$  in the rest frame of the bubble wall (where  $z$  is the coordinate normal to the surface):

$$\begin{aligned}
\dot{q} &= v_w \partial_z q + D_q \nabla^2 q - \Gamma_y [q/6 - h/8 - t/3] - \Gamma_m [q/6 - t/3 - h_0/8] \\
&\quad - 3\Gamma_{ws} [5q + 4t] - 2\Gamma_{ss} [10q + 8t] \\
\dot{t} &= v_w \partial_z t + D_q \partial_z^2 q + \Gamma_y [q/6 - h/8 - t/3] \\
&\quad + \Gamma_m [q/6 - t/3 - h_0/8] + \Gamma_{ss} [10q + 8t] \\
\dot{h} &= v_w \partial_z h + D_h \partial_z^2 h + \Gamma_y [q/6 - h/8 - t/3] - \Gamma_h(z) [(h - h_0)/8] .
\end{aligned} \tag{2.11}$$

In our past treatment of this system [10] we assumed that  $h$  and  $q - t$  (axial top number) had time independent solutions in the wall rest frame equal to the values which minimize the free energy with zero baryon number. This was tantamount to treating the dimensionless numbers  $\Gamma z_{co}/v_w$  and  $\Gamma z_{co}^2/D$  as being much larger than unity. If  $z_{co}$  is very small, as suggested by Dine and Thomas [15], then this assumption is false—especially since the rates  $\Gamma_m$  and  $\Gamma_h$  are proportional to the Higgs vev and therefore vanish at the leading edge of the wall. Eq. (2.11) allows us to deal with realistic values for  $\Gamma$ ,  $D$ , and  $v_w$ .

### 3. Solution of the rate equations

In this paper we will only use reasonable estimates of the diffusion constants  $D$  and rates  $\Gamma(z)$  in order to demonstrate the general properties of the solutions to Eq. (2.11). We will take the bubble wall profile at a transition temperature  $T$  to have the form

$$\begin{aligned}
\langle H(z) \rangle &\simeq T e^{-i\theta(z)} (1 - \tanh(z/L_w))/2 , \\
\theta(z) &= \Delta\theta (1 - \tanh(z/L_w))/2 ,
\end{aligned} \tag{3.1}$$

where  $\Delta\theta$  is a measure of the size of  $CP$  violation in the theory. It follows from (2.7) that the charge potential  $h_0$  in eq. (2.11) is given by

$$h_0 = -\frac{4}{3} v_w T^2 \frac{d\theta}{dz} \tag{3.2}$$

and that the solutions will scale linearly with  $\Delta\theta$ . For the diffusion constants we take

$$D_t = D_q = 3/T , \quad D_h = 10/T , \quad (3.3)$$

where we have used the estimates from [6] for  $D_q$  and have scaled  $D_h \simeq D_q(\alpha_s/\alpha_w)$ . For the rates (defined in Eq. (2.4) as  $\Gamma = 6\gamma/T^3$ ) we take

$$\Gamma_h \sim \Gamma_m = \langle H(z) \rangle^2 \lambda_t^2 / T , \quad (3.4)$$

and

$$\Gamma_{ws} = 6\kappa\alpha_s^4 T , \quad \Gamma_{ss} = 6\kappa \frac{8}{3} \alpha_s^4 T . \quad (3.5)$$

The steady state solution to Eqs. (2.11) was found by numerical integration. The results for the choice of parameters  $\kappa = 1$ ,  $v_w = 0.1$ ,  $L_w = 10/T$  and  $\Delta\theta = -\pi$  are given in fig. 1. The results clearly indicate that diffusion plays a significant role, since nonzero particle densities are seen to extend well beyond the bubble wall into the symmetric phase. An effect of strong sphalerons is seen in the nonzero  $q+t$  density, which vanishes as  $\Gamma_{ss} \rightarrow 0$ . Solutions to (2.11) were also obtained for a variety of wall velocities ( $0.05 \leq v_w \leq 0.3$ ), wall widths ( $10/T \leq L_w \leq 40/T$ ), and sphaleron rates ( $0.1 \leq \kappa \leq 100$ ).

The baryon density deep within the bubble ( $z$  large and negative in these coordinates) is computed by simply integrating the equation for total lefthanded fermion number (equal to  $(5q + 4t)$ , after solving algebraically for the number densities of light quarks) from far outside the bubble down to  $z_{co}$ :

$$n_B \simeq \frac{3\Gamma_{ws}}{v_w} \int_{z_{co}}^{\infty} dz (5q(z) + 4t(z)) . \quad (3.6)$$

As discussed in the introduction, the value of  $z_{co}$  is controversial, and a large value (corresponding to small  $\phi_{co}$ ) suppresses baryon number when diffusion is ignored. In fig. 2 we present the final baryon asymmetry  $n_B/s$  of the universe as a function of  $z_{co}$  for several values of  $\kappa$ , as well as the solution for  $\kappa = 1$  when all the diffusion constants are taken to be very small,  $D_q = D_h = 0.1/T$ . Apparently spontaneous electroweak baryogenesis can quite easily account for the observed asymmetry  $n_B/s \sim 10^{-10}$  with  $CP$  violation at the  $\Delta\theta \sim 10^{-2} - 10^{-3}$  level.

There are several remarkable features to fig. 2: First, it illustrates the Dine-Thomas sensitivity to the value of  $z_{co}$  when diffusion is ignored, but the sensitivity is practically eliminated when diffusion is taken into account. A second amusing feature is that the final baryon asymmetry turns out to be approximately independent of  $\kappa$  in the range examined. Ignoring the effects of strong sphalerons one would have found  $n_B \propto \kappa$ , but as pointed out by Giudice and Shaposhnikov [18] the left handed baryon number is driven to zero by strong sphalerons in the limit that the thermal fermion masses are neglected. We find numerically that the baryon number with  $z_{co} = 3.0L_w$  is inversely proportional to  $\Gamma_{ss}$  for  $0.5 \lesssim \kappa \lesssim 100$ , cancelling the linear  $\kappa$  dependence for electroweak sphalerons. For  $\kappa \lesssim 0.1$ , we find  $n_B$  to be insensitive to  $\Gamma_{ss}$  and proportional to  $\kappa$  (an intuitive understanding of this number is that strong sphalerons are irrelevant when the timescale  $1/\Gamma_{ss}$  is much longer than the typical time  $D/v_w^2$  that the diffusing fermions spend in the symmetric phase.)<sup>3</sup>

We conclude this section by noting that solutions to Eq. (2.11) with  $\kappa = 1$  and a variety of wall velocities and wall widths reveal that the baryon asymmetry one creates is not extremely sensitive to either  $v_w$  or  $L_w$ . We find that for  $L_w = 10/T$  and  $v_w$  ranging from 0.05 to 0.3, the baryon asymmetry changes by no more than a factor of 2; for a broader wall with  $v_w = 0.1$  and  $L_w = 40/T$ ,  $n_B$  was decreased by a factor of  $\sim 5$ . When we artificially set the diffusion constants and  $\Gamma_{ss}$  to be very small and assume  $\phi_{co} = 14\alpha_w T/g$ ,  $\kappa = 1$ , we reproduce our earlier results which neglected both diffusion and strong sphalerons<sup>4</sup>.

## 4. Outlook

We conclude that a dynamical treatment of spontaneous electroweak baryogenesis with finite interaction rates, nonzero diffusion constants, and strong sphaleron effects eliminates

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<sup>3</sup> We have not presented data for  $\kappa > 5.0$  since the neglected  $m_t$  and  $\alpha$  corrections to the relation  $n \simeq k\mu T^2/6$  discussed in [18] are expected to become more important as  $\kappa$  gets larger. These effects are expected to further enhance the baryon asymmetry. Note however that even neglecting these effects one finds a sizable baryon asymmetry in our dynamical calculation, unlike in the equilibrium calculation of [18].

<sup>4</sup> Our review [2] contains a typographical error in Eq. 37: the right side of Eq. 37 should include the factor  $(\mathcal{N}/.33)$  instead of  $(\mathcal{N}/.1)$

or moderates large suppression factors found in an equilibrium calculation. Our work indicates that the EWB scenario remains a viable explanation for the baryon asymmetry of the universe. Furthermore, the results appear not to be very sensitive to the bubble wall velocity or width, to the point in the wall where anomalous baryon violation becomes suppressed, or to the sphaleron rate parameter  $\kappa$  for a broad range of these parameters. This is encouraging, since these quantities are poorly known even in specific models. On the other hand, our results are rather sensitive to the values of the diffusion lengths and interaction rates, which we have only estimated. These quantities are computable from known physics, and this should be done before making firm conclusions. Corrections of order  $m_t^2$  and  $\alpha$  to the relationship between density and charge potential should also be included, particularly for large  $\kappa$ .

A more complete treatment would also include the effects of hypercharge screening [19,7]. Screening is known not to affect the spontaneous baryogenesis in equilibrium calculations, since three quark doublets are drawn in for every anti-lepton doublet, so that there is no net effect on weak sphalerons. In a dynamical calculation one might expect the anti-leptons to screen more efficiently since they have a longer mean free path, in which case the baryon number produced would only be enhanced.

Finally, we note that diffusion will be even more important for spontaneous baryogenesis in the minimal supersymmetric model [13], due to the contributions from charginos and neutralinos which have much longer mean free paths than quarks. In addition, strong sphaleron suppression will be absent due the combinations of charges that bias baryon number in that model.

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## Figure Captions

- Fig. 1. Time independent particle number densities normalized to the entropy as a function of position  $z$ , in the rest frame of the expanding bubble wall. The parameters used were  $v_w = 0.1$ ,  $L_w = 10/T$ ,  $\kappa = 1$ , and  $\Delta\theta = -\pi$ ; the  $D$  and  $\Gamma$  coefficients used are given in Eqs. (3.3), (3.4), (3.5). The densities  $h$ ,  $q - t$ , and  $q + t$  refer to Higgs number,  $t_L + b_L - t_R$ , and  $t_L + b_L + t_R$  respectively. The midpoint of the bubble wall is at  $z = 0$ .
- Fig. 2. The final baryon to entropy ratio of the universe for several values of  $\kappa$ , plotted as  $\text{Log}_{10}[(n_B/s)(g_*/100)(-\pi/\Delta\theta)]$  versus  $z_{co}$ , the point where weak sphalerons become exponentially suppressed.  $z_{co} = -0.5L_w$  corresponds to  $\phi_{co} \simeq 14\alpha_w T/g$ , while  $z_{co} = 3.0L_w$  corresponds to  $\phi_{co} \simeq \alpha_w T/g$ . The dashed line is the solution to Eq. (2.11) assuming very small diffusion constants, illustrating the Dine-Thomas effect.

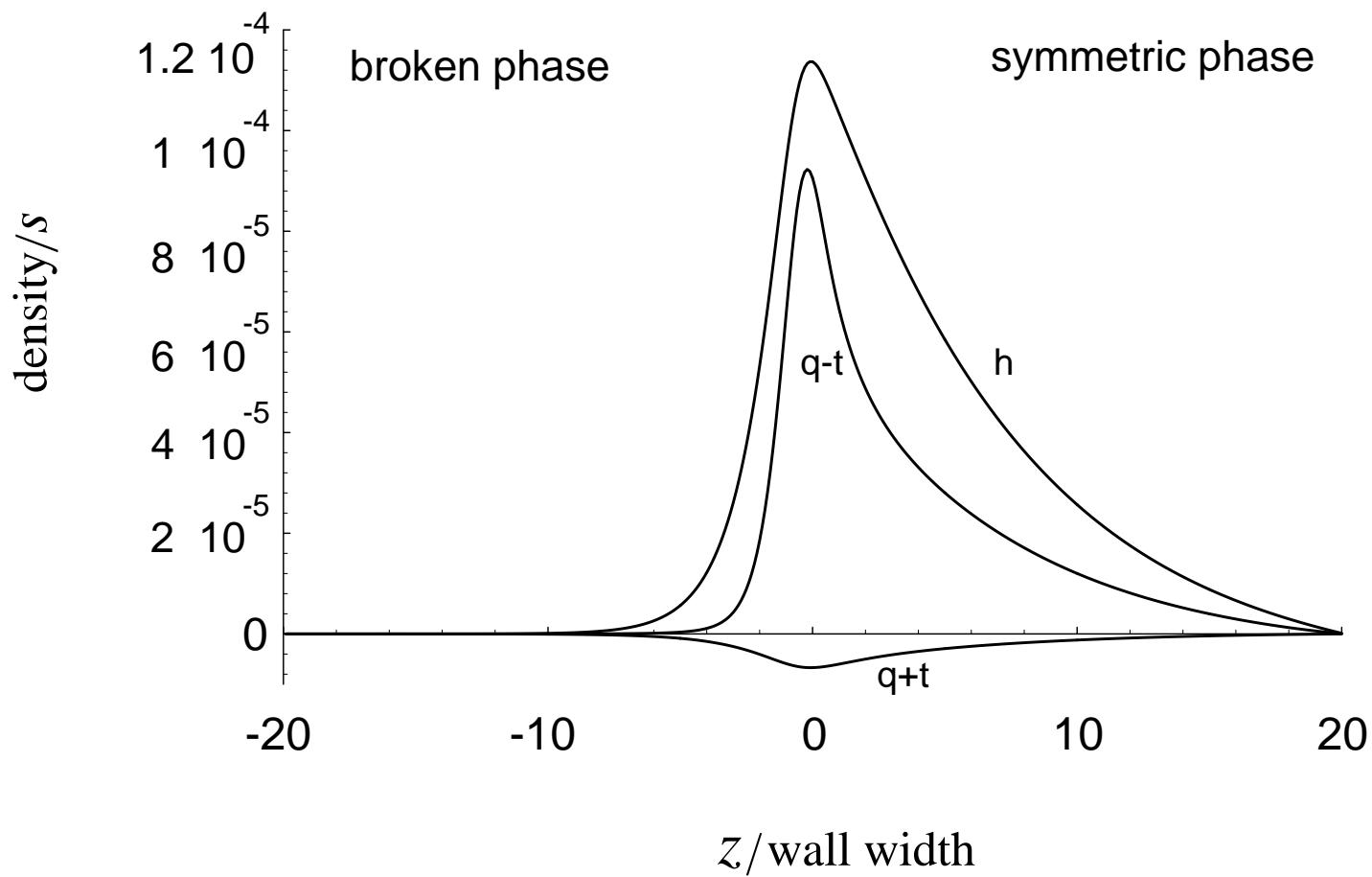


Figure 1

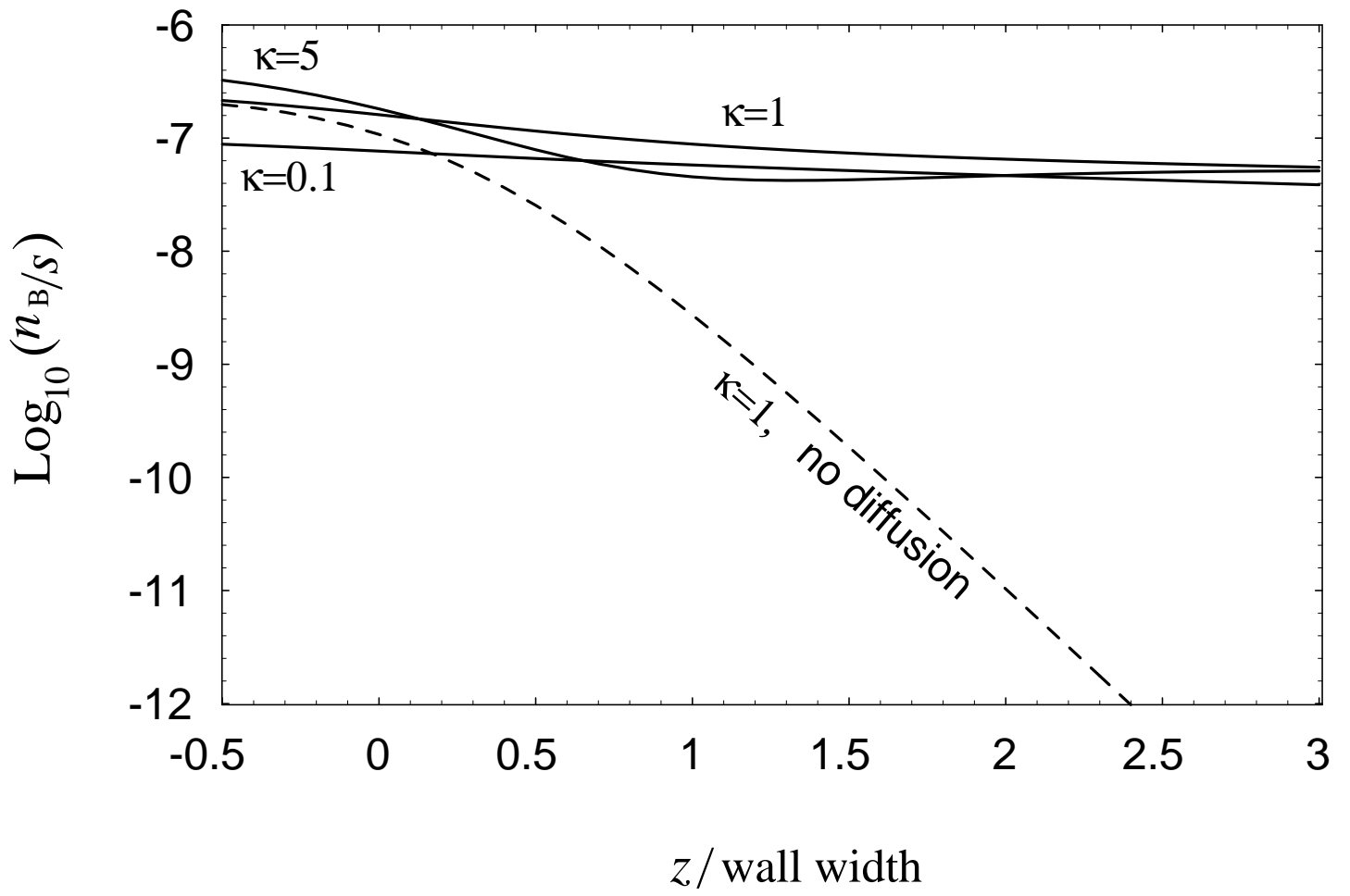


Figure 2